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MINIMUM DOMINATING SET OF SOME GRAPHS

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Abstract. A set D ⊆ V of vertices in a graph G = (V, E) is called a dominating set if every vertex v ∈ V (G) is either an element of D or is adjacent to an element of D [4].The domination number γ(G) is the minimum cardinality of the dominating set of G. This article embarks on an exploration of the domination number of a graph G. Additionally, it delves into the precise determination of minimum dominating set for certain well-known graphs.

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Key Words: Dominating set, Domination number.

1. INTRODUCTION

All the graphs considered here are finite, undirected with no loops and multiple edges. A graph G is an ordered triple G = (V (G); E(G); ψ(G)) consisting of a non-empty set V (G) of vertices, a set E(G) of edges disjoint from V (G)and an incidence function ψ(G) which associates with each element of E(G),an unordered pair of vertices (not necessarily distinct) of G. If two vertices are incident with a common edge, then that vertices are adjacent. Otherwise, they are non-adjacent. If two edges are incident with a common vertex, then that edges are adjacent. Otherwise, non-adjacent. The degree of a vertex of a graph is the number of edges incident to the vertex, with loops counted twice. The degree of a vertex v is denoted by deg(v) . The maximum and minimum degree of a graph is denoted by Δ(G) and δ(G) respectively. N (v) and N [v] denote the open and closed neighbourhoods of a vertex v respectively. The vertex with zero degree is called an isolated vertex. A vertex with degree one is called an end vertex or pendant vertex [5].

A set D ⊆ V of vertices in a graph G = (V, E) is called a dominating set if every vertex v ∈ V (G) is either an element of D or is adjacent to an element of D[4].The domination number γ(G) is the minimum cardinality of the dominating set of G. For instance, consider the Peterson graph P, whose minimum dominating set illustrated in Figure 1.

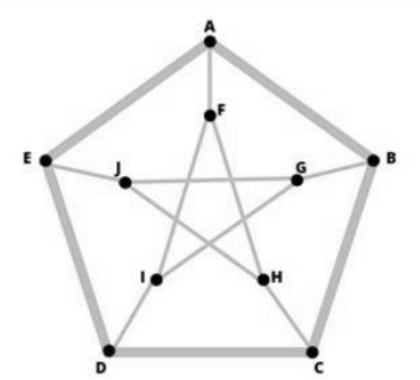

Figure 1. In this graph P, D = {C, F, J} is a dominating set and no subset of the vertex set with cardinality less than three has this property so γ(P) = 3.

In this article we discuss the determination of minimum dominating set for certain well-known graphs.

2. MINIMUM DOMINATING SET OF SOME GRAPHS

A walk in a graph is a finite non-null or non-empty sequence $w = v_0e_1v_1e_2v_2 \cdot\cdot\cdot e_kv_k$ whose terms are alternatively vertices and edges so that for $1 \leq i \leq k$, the ends of e_i are v_{i-1} and v_i. We say that w is a walk from v_0 to v_k or w is a (v_0, v_k) walk. A walk in which every vertex is distinct (hence edges are distinct) is called a path [5]. a cycle C is a finite non-null or non-empty sequence $w = v_0e_1v_1e_2v_2 \cdot\cdot\cdot e_kv_0$ where $v_1 \ldots v_{k-1}$ are distinct.

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Theorem 2.1. For a path Pn, γ(Pn) = ⌈n/3⌉.

Proof. Consider a path Pn, with V (Pn) = {v1, v2, · · ·, vn}. For i = 2, 3, · · ·, n−1 each vertex vi is adjacent with v_{i-1} and v_{i+1}. It can be easily observed that

γ(P1) = γ(P2) = 1, for n ≥ 3 consider the following cases and following subset of V (Pn).

Case 1: Assume n ≡ 1 mod 3, D = {v_{3k+2}, v_n} where 0 ≤ k <⌈n/3⌉ − 1.

Case 2: Assume n ≡ 0, 2 mod 3, D = {v_{3k+2}} where 0 ≤ k <⌈n/3⌉. in both cases D dominates Pn, D is a dominating set. No set with cardinality less than |D| have this property. Therefore γ(Pn) = ⌈n/3⌉.

Theorem 2.2. For a Cycle Cn, γ(Cn) = ⌈n/3⌉.

Proof. Consider a path Cn, with V (Cn) = {v1, v2, · · ·, vn}. For i = 2, 3, · · ·, n−1 each vertex vi is adjacent with v_{i-1} and v_{i+1}. For n ≥ 3 consider D = {v_{3k+1}} where 0 ≤ k <⌈n/3⌉. D dominates Cn, D is a dominating set. No set with cardinality less than |D| have this property. Therefore γ(Cn) = ⌈n/3⌉. □

A graph G is complete if every pair of distinct vertices is joined by an edge and G is simple. A complete graph on n vertices is denoted by Kn. From the definition of domination number, we have the following result.

Theorem 2.3. For a complete graph Kn, γ(Kn) = 1.

Proof. Let V be the vertex set of Kn. Any singleton subset of V is a dominating set of Kn. Hence γ(Kn) = 1. □

For a wheel graph, Wn, is a graph obtained by connecting a single vertex to all vertices of a cycle graph Cn−1.

Theorem 2.4. For a wheel graph Wn, γ(Wn) = 1.

Proof. Let v be the vertex of Wn having degree n − 1. The set {v} is a dominating set for Wn. Hence γ(Wn) = 1. □

From the above results it is clear that, if G is a graph with n vertices. And Δ(G) = n − 1, then γ(G) = 1.

Theorem 2.5. Let Km,n be a complete bipartite where n, m ≥ 2. Then γ(Km,n) =2.

Theorem 2.6. For a star graph K1, n, γ(K1,n) = 1.

A helm graph is a graph that is created by attaching a pendant edge to each vertex of an n-wheel graph’s cycle, and is denoted by Hm, where m ≥ 4.

Theorem 2.7. Let Hm be a helm graph. Then the domination number of graph Hm is m.

Proof. Let v0 denote the vertex in helm graph such that its degree is equal to m, and let v1, v2, · · ·, vm represent the vertices in the helm graph, each with a degree of 3. Also, u1, u2, · · ·, um represent the pendant vertices of the helm graph Hm (See Figure 2). It can be easily verified that D = {v1, u2, u3, · · ·, um}is a dominating set of Hm, which is minimum. Therefore, γ(Hm) = m.

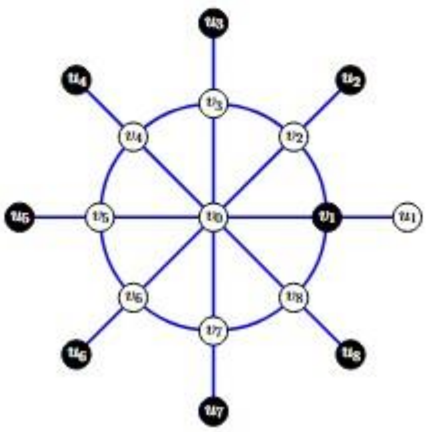

Figure 2. The Helm graph H8, dominating set D = {v1, u2, u3, · · · , u8} and γ(H8) = 8

A Coconut tree CT (m, n) is the graph obtained from the path Pm by appending ’n’ new pendant edges at an end vertex of Pm.

Theorem 2.8. For any coconut tree graph CT (m, n), the domination number is

1 + ⌈(m−2)/3 ⌉, where m ≥ 2, n ≥ 2.

Proof. Let G = CT (m, n) be a coconut tree on m + n vertices with m + (n − 1) edges and let D be a minimum dominating set of graph G. By definition of coconut tree, the graph is obtained from the path Pm by appending n new pendant edges at an end vertex v1 (say) of Pm. Clearly the vertex v1 must be included in any minimum dominating set, since Δ[CT (m, n)] = deg(v1). In G, v1 dominate all pendant vertices attached with v1. Therefore, the domination number of G is 1 + ⌈(m−2)/3 ⌉. □

A diamond snake graph Dn is a connected graph obtained from a path P of length n with each edge e = (u, v) in P replaced by a cycle of length 4 with u and v as nonadjacent vertices of the cycle.

Theorem 2.9. For any diamond snake graph, γ(Dn) = n + 1.

Proof. Let u1, · · ·, un, v1, · ·, vn+1, w1, · · ·, wn be the vertices of diamond snake graph Dn (see Figure 3). Consider D = {v1, · · ·, vn+1}. Now D form a dominating

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set. It can be easily verified that no set with less than |D| vertices cannot form dominating set. Therefore γ(Dn) = n + 1. □

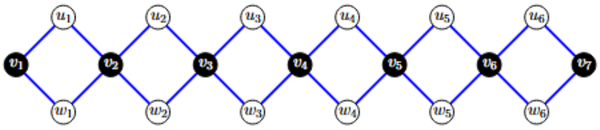

Figure 3. The Diamond Snake Graph D6, dominating set D ={v1, v2, · · · , v7} and γ(D6) = 7

The ladder graph Ln is the graph obtained by taking the Cartesian product of Pn with P2.Also Ln has 2n vertices and 3n-2 edges.

Theorem 2.10. For a ladder graph L_n, n ≥ 2, $\gamma(L_n)$ =

$$\begin{cases} \frac{n+1}{2} \; if\ n\ is\ odd \\ \frac{n+2}{2} \; if\ n\ is\ even \end{cases}$$

Proof. Let {u1, u2, · · · , un, v1, v2, · · · , vn} be the vertices of the ladder graph L_n. For L_2, {u1, v2} be a dominating set and for L3, {u1, v3} be a dominating set. And these are minimum. For n ≥ 4 depending upon the number of vertices of Ln we consider the following cases and subset of vertex set of Ln.

Case 1: For n ≡ 0, 1 mod 4, S = $\{u_{4k+1}, v_{4k+3}, u_n\}$ where $0 \leq k < \left\lfloor \frac{n}{4} \right\rfloor$

Case 2: For n ≡ 2, 3 mod 4, S = $\{u_1, v_{4k+3}, u_{4k+5}, v_n\}$ where $0 \leq k < \left\lfloor \frac{n}{4} \right\rfloor$.

Now S forms a dominating set. It can be easily verified that no set with less than |S| vertices

Can not form a dominating set. So $\gamma(L_n)$ =

$$\begin{cases} \frac{n+1}{2} \; if\ n\ is\ odd \\ \frac{n+2}{2} \; if\ n\ is\ even \end{cases}$$

3. CONCLUDING REMARKS

This paper has explored the concept of domination number in graphs, a fundamental parameter in graph theory with far-reaching implications in computer science, operations research, and network optimization. Through a comprehensive review of existing literature and novel results, we have shed new light on the domination number of some well-known graphs.

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